Escape to a New Dimension – A Journey through Space with a Square, a Cube, and a Tesseract

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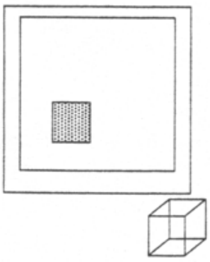
A play in "tesser"-acts inspired by the literary genius of Edwin Abbott and the graphical insights of Thomas Banchoff

This is a test of your imagination. Put yourself in the following scene:

You are a cube. One day, while watching your favorite movie, "Sigma, the Life of a Square in Flatland," the DVD suddenly stops. The image of Sigma, the square, stays frozen on the screen. As you get up to investigate, you hear Sigma speak.



Sigma: What happened? I can't move!
Cube: The DVD stopped.
Sigma: Who's that talking to me?
Cube: I am. I'm a cube.
Sigma: I don't see you. You're not above, below, left, or right of me.

The square has only one eye on one of his sides and can only see in the two-dimensional space in which he lives.

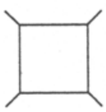
Cube: No, I'm not on the screen. You have to look OUT of the screen.
Sigma: NONSENSE! Everyone knows that there are only TWO dimensions. How can anyone see outside of them?

Cube: You mean to tell me that you can only see in the plane you are situated in?
Sigma: Of course, that's all anyone can see. What kind of joke is this? Where are you hiding?
Cube: Honest, I am a cube, and I'm outside your plane.
Sigma: What's a cube?
Cube: Well, let's say that I'm a three-dimensional square.
Sigma: No way! There is no such thing. Is that you, Uncle Elmo? I thought you were dead. This is the best practical joke that you've pulled yet. How did you make yourself invisible?
Cube: I am NOT your Uncle Elmo. Just because you can't see me doesn't mean that I can't exist.
Sigma: OK, I'll play along. Explain to me what you look like.
Cube: I've already told you. I'm a three-dimensional square.
Sigma: Cut out the higher-dimension jargon and describe yourself in terms that I can understand!
Cube: You mean in two-dimensional terms?
Sigma: Of course! Stick to reality.
Cube: OK, let me try a few explanations.
Sigma: You've got a captive audience.

THE CUBE DESCRIBED AS SEEN FROM A *FACE*

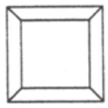
You take a marker and proceed to draw on the screen.

Cube: Do you see that?
Sigma: Yes, it looks like me.
Cube: All right, do you see these lines?



You draw segments radiating from each vertex.

Sigma: It looks like me with spiked hair!



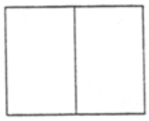
Cube: Now, if I draw another square around it, this is what I'd look like.
Sigma: So you're a square inside of a square?
Cube: No, but if you could see me, that is what I'd look like. What would *you* look like if someone were to look at you from the side?

Sigma: If you could see through me, I'd look like a segment with a shorter segment behind me.
Cube: Well, this is the same kind of effect.
Sigma: I'm still a little confused.
Cube: Well, let's try to look at it from a different angle.
Sigma: All right, this is getting interesting.

THE CUBE AS SEEN FROM AN *EDGE*

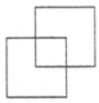
Cube: What if someone were to look at you directly from a vertex?

Sigma: I'd look like two short segments stuck together.

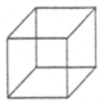


Cube: OK, if you looked at me from a corner, I'd look like this.
Sigma: Two skinny rectangles stuck together?
Cube: Yes. It's the same situation.
Sigma: So you have corners and sides just like me?
Cube: Yes, but I also have an additional type of corner because I am three-dimensional.
Sigma: This is starting to sound like the Twilight Zone.

THE CUBE DESCRIBED AS SEEN FROM A *VERTEX*



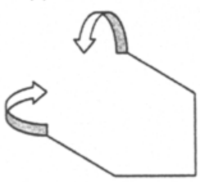
Cube: Now, take a square and slide a duplicate copy of it at a 45-degree angle for a short distance.
Sigma: So now you have two squares.
Cube: Connect the vertices of the original square to the corresponding vertices of the copy.

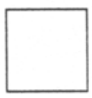


Sigma: Now you've got a hexagon with some segments drawn inside that cross each other.
Cube: Well, they don't really cross; they just LOOK like they do.
Sigma: What do you mean? It's obvious that they cross each other.
Cube: Not if the copy of the original square was really moved in a third dimension, instead of a 45-degree angle.
Sigma: Once again, you've lost me with that extra dimension.

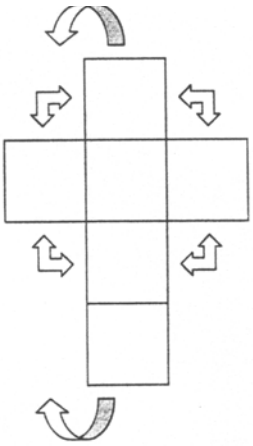
THE CUBE BUILT FROM TWO-DIMENSIONAL MATERIALS

Cube: OK, let's try to put this in two-dimensional terms.
Sigma: That's what you're *supposed* to be doing!
Cube: Now, how would I build a square?
Sigma: You could take a line segment and fold it into four equal sides. Just make sure that they all meet at right angles.





Cube: I can be constructed from six congruent squares. If I were flattened out, I would look like this.



Sigma: So you're a religious symbol?
Cube: No. Now, if you fold up the figure at right angles, you'll have a cube.
Sigma: That's impossible! You can't get any of those edges together without stretching segments!
Cube: Sure you can. Just fold them up, outside of the plane.
Sigma: Here we go again.
Cube: Let's try one more method.

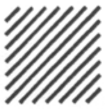
THE CUBE DESCRIBED AS A STACK OF SLICES CUT FROM A FACE

Cube: Do you agree that I can think of a square as being built from thick segments?
Sigma: Sure, if you stacked up enough congruent segments, it would look like a square.
Cube: A cube is just a stack of congruent squares.

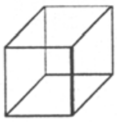
Sigma: So you are a tower of squares stacked end to end? I can visualize that.
Cube: NO! The squares are stacked so that the corresponding sides are next to each other.
Sigma: Don't tell me, let me guess. They're stacked in a different dimension?

THE CUBE DESCRIBED AS A STACK OF SLICES CUT FROM AN EDGE

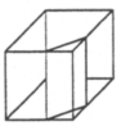
Cube: Touché! OK, what if you were sliced into thin strips starting from a corner?



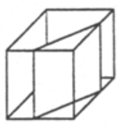
Sigma: First, there'd be a slice that looks almost like a point. Then the slices would look like longer and longer segments until you got to the diagonal in the middle. The rest of the slices would look the same, only in reverse order.



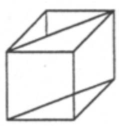
Cube: Good. If I were sliced from a corner, the first slice would look like this.
Sigma: A line segment?



Cube: Now as I continued, the slices would start to look like . . .
Sigma: Thin rectangles?
Cube: Yes. Notice that they are all the same height.



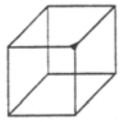
Sigma: But they get wider as you continue!
Cube: Yes, just like your slices get longer as you continue toward the middle.
Sigma: So, you get a fat rectangle in the middle?



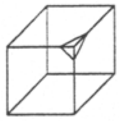
Cube: Exactly. You're starting to catch on!
Sigma: And then the rest of the slices would look the same but in reverse order?
Cube: YES! And what would the last slice look like?
Sigma: Just like the first: a segment. Wow, I'm starting to follow your analogy! What would happen if the slices were made from one of your "three-dimensional" corners?

THE CUBE DESCRIBED AS A STACK OF SLICES CUT FROM A VERTEX

Cube: If you think you're ready, this last slice sequence is the most difficult.
Sigma: No problem. I've got the hang of it now.

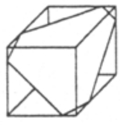


Cube: The first slice would look like a point.
Sigma: Easy enough.
Cube: The next few would look like equilateral triangles.

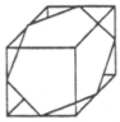


Sigma: I guess I can follow that. And they will increase in size like before?
Cube: Yes, but as we near the "middle" this time . . .

Sigma: The corners get cut off and you have hexagons? But as the corner cuts get larger, it's going to look like a regular hexagon.
Cube: Yes. Guess where.



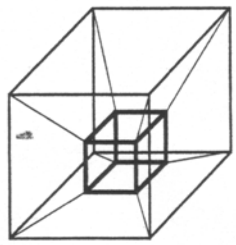
Sigma: In the middle?
Cube: EXACTLY!
Sigma: Supposing that this is all true—then the rest of the slices are the same, only in reverse?



Cube: Well, reversed and upside down.
Sigma: What? Upside down!
Cube: In other words, you reverse everything.
Sigma: Until you get a point?
Cube: Yes, there you go. Understand?
Sigma: Yes, I guess. But I don't see why anyone needs three dimensions anyway. If you can't see it, how do you know it exists?
Tess(eract): Excuse me for interrupting, but why stop at three dimensions?
Sigma: Who's that?
Cube: Ditto!
Tess: I am a hypercube, a four-dimensional cube, but I prefer "tesseract" since I've quit drinking caffeine.
Sigma: Oh, no. You mean things can get even *more* demented?
Cube: You better let me handle this. FOUR DIMENSIONS?!
Tess: Yeah, I've been watching you explain things to Sigma. The same analogies can be used for you to understand four space.

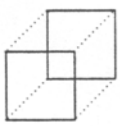
A TESSERACT AS SEEN BY A CUBE LOOKING OUT FROM THREE-DIMENSIONAL SPACE

Cube: How so?

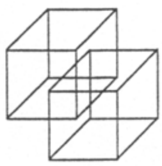


Tess: Remember that you told Sigma that you look like a square inside of a square?
Cube: Sure. And I suppose you look like a cube inside of a cube?
Tess: Yes. Imagine you and a copy of you that is moved in a direction you can't see. Now, connect the corresponding vertices.

A TESSERACT AS A CONSTRUCTION OF EIGHT CUBES

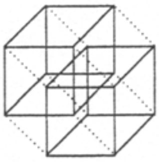


Cube: OK, how can I explain this to Sigma?
Tess: Well, you already told him that a cube can be thought of as duplicating a square, sliding it off a bit, and then connecting the corresponding vertices.



Cube: Yeah, he bought it.
Tess: Do the same thing again.
Cube: You mean duplicate, slide, and connect?
Tess: Yes, exactly.
Cube: But this is two-dimensional!

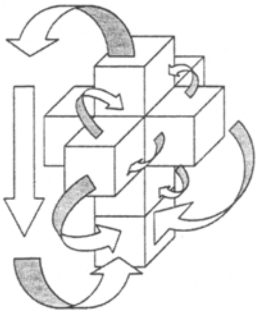
Tess: No, it's a two-dimensional representation.



Cube: Oh, each time you slide, it is really in a different dimension?
Tess: Yes!
Cube: So all those other analogies that I used for Sigma will also work for me.

A TESSERACT AS BUILT FROM EIGHT CUBES

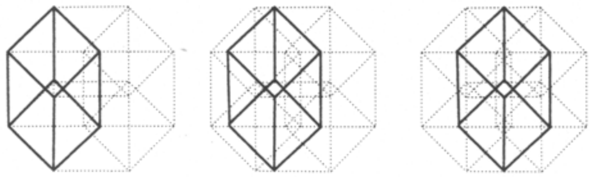
Tess: So, if a square is formed by folding a segment that is divided into four equal segments, and a cube is formed by folding six congruent squares and connecting them, . . .



Cube: Then a tesseract is formed by folding *eight* cubes and connecting them?
Tess: Yes.
Cube: Cool!

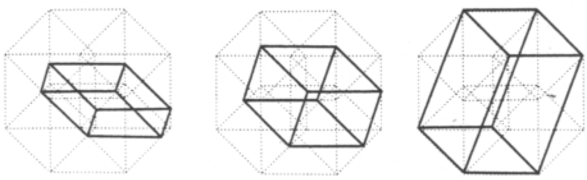
SLICES OF A TESSERACT PARALLEL TO A CUBE

Tess: Let's talk about slices of a tesseract.
Cube: OK, since I can be sliced in a number of different ways, I assume that you can, too! Probably four, since I can be sliced three ways.
Tess: Yes.
Cube: When I am sliced parallel to one of my square faces, a series of squares is created. So if you are sliced parallel to one of your cubic faces, you should get a series of cubes!
Tess: Right again!



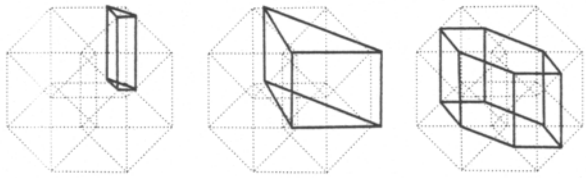
SLICES OF THE TESSERACT PARALLEL TO A SQUARE

Cube: If you are sliced from a square face it would be more complicated, wouldn't it?
Tess: Yes.
Cube: When I am cut parallel to an edge, I become a series of rectangles with the widest rectangle at the center. So when you are sliced parallel to a square, I guess that you would be a series of rectangular prisms, with the widest rectangular prism at the center.
Tess: Correct.



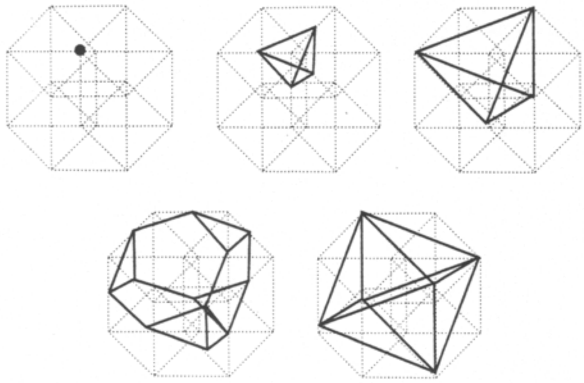
SLICES OF A TESSERACT PARALLEL TO A SEGMENT

Cube: And when I am sliced with respect to a point, the first few slices look like triangles, then the vertices get cut off so that there are hexagons, with a regular hexagon in the middle. So when you are sliced with respect to a segment, I suppose that, initially, the first few slices will be triangular prisms, then hexagonal prisms, with a regular hexagonal prism in the middle.
Tess: And then the situation reverses itself as you continue, of course.



SLICES OF A TESSEARCT FROM A VERTEX

Cube: Now I am out of ideas. What would the slices look like if they were cut parallel to a vertex?
Tess: I admit, this one will be a little bit hard to visualize.
Cube: Try me!
Tess: It will start out as a regular tetrahedron. Then the corners will be cut so that it will be a semiregular solid with four triangular and four hexagonal sides. In the middle, the figure will actually be a regular octahedron.
Cube: You don't say! SIGMA!



Sigma: It's obvious! Just follow the analogies. It also helps to sketch the figures. I can follow all of this because all of the drawings are in two dimensions.
Cube: Wait a minute! You mean it's easier if I go DOWN a dimension?
Tess: Yeah, be flexible!
Mystery Voice: Hey, speaking of flexible. I am a five-dimensional cube. I couldn't help overhearing your conversation and . . .

Et cetera . . .